

## KEY FEATURES OF POLYNOMIAL GRAPHS

**Zeroes (or x-intercepts):** pts. where the graph crosses the x-axis ( $\neq, 0$ )

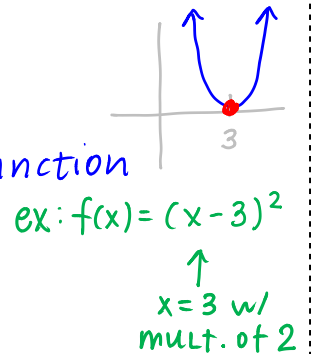
How to find them:

1. Set  $y=0$ , factor then solve

\*\* A polynomial of  $n$  degree has at most  $n$  zeroes. \*\*

**Multiplicity:** how many times a "zero" appears in a function

- If the multiplicity is EVEN, the graph "kisses" the x-axis at that point.
- If the multiplicity is ODD, the graph crosses the x-axis at that point.



**Y-intercepts:** where the graph crosses the y-axis

How to find them:

1. plug in 0 for  $x$  & solve for  $y$   
( $y$ -int. is always the constant in the function)

**End Behavior:** what the graph does as  $x$  approaches  $\infty$  &  $-\infty$

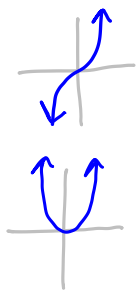
How to find it:

1. Identify the degree and the leading coefficient

- If the degree is odd, the end behavior goes in opp. directions
- If the degree is even, the end behavior goes in the same direction

2. Plug in a really big # number to identify the end behavior as  $x$  approaches  $+\infty$

3. Based on if the degree is even or odd, use the right end behavior to determine the left end behavior (what happens as  $x$  approaches  $-\infty$ )



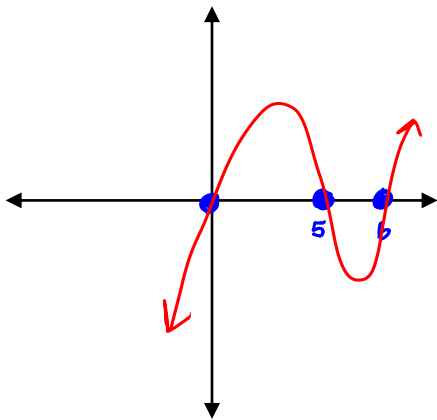
**Notation:**

- $\lim_{x \rightarrow \infty} f(x)$ : limit of  $f(x)$  as  $x$  approaches  $+\infty$  (right side behavior)
- $\lim_{x \rightarrow -\infty} f(x)$ : limit of  $f(x)$  as  $x$  approaches  $-\infty$  (left side behavior)

## SKETCHING GRAPHS

Use the following functions to answer the questions below. Then, sketch a graph of the polynomial.

1.  $f(x) = x^3 - 11x^2 + 30x = x(x^2 - 11x + 30)$   
 $f(x) = x(x-6)(x-5)$



What is the leading coefficient of the function?

1 (right end behavior  $\uparrow$ )

What is the degree of the function?

3 (opp. end behavior)

Is the degree of the highest degree term ODD or EVEN?

odd (left end behavior  $\downarrow$ )

What are the zeros of the function?

$$x = 0, 5, 6$$

Multiplicity?

$\uparrow \uparrow \uparrow$   
 1 1 1

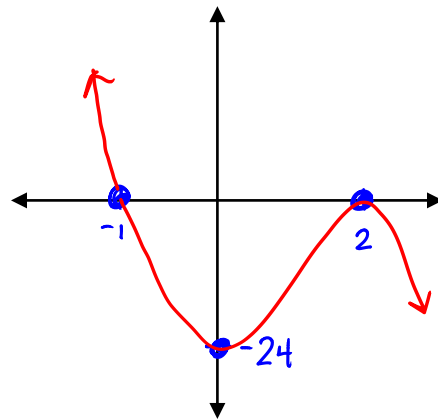
What is the y-intercept of the function? 0

What will the end behavior of the function be?

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

2.  $g(x) = -6(x-2)^2(x+1)^3$



What is the leading coefficient of the function?

-6 (right end behavior  $\downarrow$ )

What is the degree of the function?

5 (opp. end behavior)

Is the degree of the highest degree term ODD or EVEN?

odd (Left end behavior  $\uparrow$ )

What are the zeros of the function?

$$x = -1, 2$$

Multiplicity?

$\uparrow \uparrow$   
 3 2

What is the y-intercept of the function? -24

$$-6(-2)^2(1)^3 = -24$$

What will the end behavior of the function be?

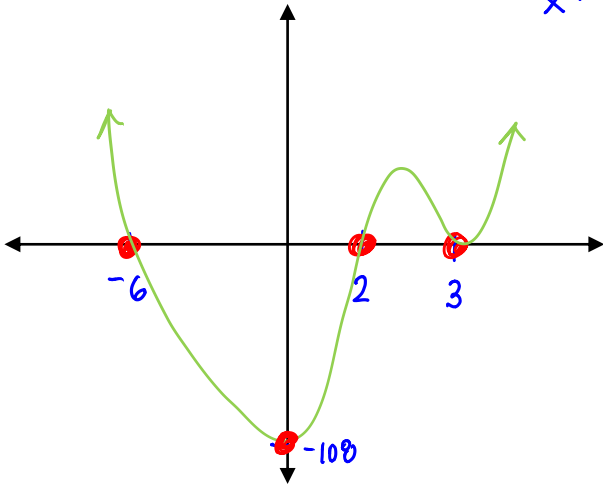
$$\lim_{x \rightarrow \infty} g(x) = -\infty$$

$$\lim_{x \rightarrow -\infty} g(x) = \infty$$

**You Try!**

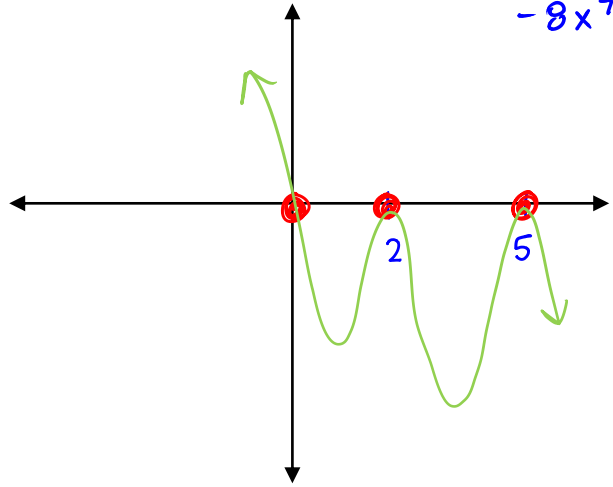
For the following functions, find the zeros, y-intercepts and end behavior of each. Then sketch a graph and label all points of importance.

3.  $h(x) = (6+x)(x-2)(3-x)^2 = x \cdot x \cdot (-x)^2$   
 $x^4$



Leading term:  $x^4$   
 Degree: 4  
 Zeros w/Multiplicity: -6(1) 2(1) 3(2)  
 Y-intercept: -108  
 End Behavior:  $\lim_{x \rightarrow \pm\infty} h(x) = \infty$

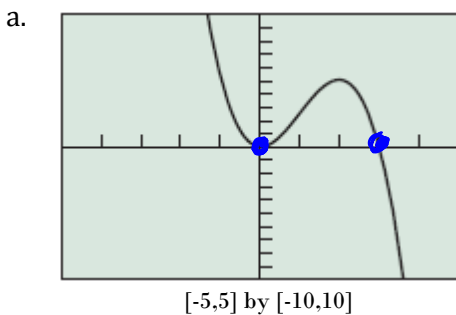
4.  $j(x) = -2x^3(x-2)^2(2x-10)^2 \Rightarrow -2x^3 \cdot x^2 \cdot (2x)^2$   
 $-8x^7$



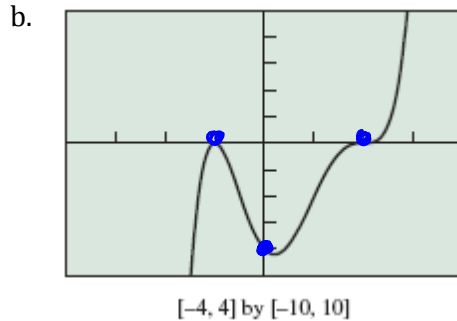
Leading term:  $-8x^7$   
 Degree: 7  
 Zeros w/Multiplicity: 0(3) 2(2) 5(2)  
 Y-intercept: 0  
 End Behavior:  $\lim_{x \rightarrow \infty} j(x) = -\infty$   
 $\lim_{x \rightarrow -\infty} j(x) = \infty$

**CREATING EQUATIONS FROM A GRAPH**

Write a possible function **IN FACTORED FORM** for the following graphs.



Equation:  $f(x) = -x^2(x-3)$



Equation:  $g(x) = 2(x+1)^2(x-2)$